TM-1063
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Fermilab

DESIGN CONSIDERATIONS IN VAPOR PRESSURE THERMOMETRY

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ABSTRACT

Proper VPT design identifies the upper limit of the vapor pressure bulb region, the beginning of the gas bulb region, and fixes the liquid level of the bulb below a level consistent with quick response over the entire operating range. These points are determined by charge pressure and the ratio of, and details of connections between, the warm and cold volumes. Knowledge of the volume ratio and the charge pressure can be used to specify the temperature range of the VPT, and leads to increased utilization as a gas bulb thermometer.

GAS BULB CURVE

Determining the VPT/gas bulb thermometer transition requires plotting a curve of P vs. T, for a given geometry, in the gas region and locating the intersection with the vapor pressure curve. Generating that curve requires knowledge of the following parameters: charge pressure P_I, charge temperature T_I, warm volume V_w, cold volume V_c, and the length of capillary tubing experiencing a thermal gradient from ambient temperature to the temperature at the sensing bulb T_c. The capillary length L_c, is only the portion experiencing a gradient; i.e., the distance from the point on a heat exchanger where the tube enters a vacuum jacket to the point it connects to the sensing bulb. The tube outside the vacuum jacket is included in V_w. Knowing P_I and T_I defines the initial system density ρ_I, which can be found using the NBS tables of thermophysical properties for the charging fluid. The total system volume, V_T, is known and the system mass, M_s, follows:

$$M_s = \rho_I \cdot V_T$$

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As the bulb cools the measured pressure P_m decreases. It is important to determine how mass is distributed in each part of the system. Knowing P_m and the ambient temperature defines the warm density ρ_w. M_w, the mass in the warm region, follows:

$$M_w = \rho_w \cdot V_w$$

Finding the mass in the capillary tube M_{cap}, is more involved. A plot of thermal conductivity vs. temperature is shown in Figure 1. Integrating the curve from ambient temperature to T_c yields a value of $K(T_c)$ (watts/cm). L_c can be divided into i segments such that $\sum_o^i L_i = L_c$. The thermal conductivity over segment i is λ_i, and the temperature difference is ΔT_i. The length of the segment L_i then follows as

$$L_i = \frac{\lambda_i \ L_c \ \ T_i}{K(T_c)}$$

Note that L_i is independent of capillary cross section. Each segment L_i has a temperature which is the average value over ΔT_i. Each ΔT_i has a unique ρ_i since P_m is constant for the tube. Each segment's mass may then be calculated, assuming the area of the hole in the tube is known

$$M_i = L_i \cdot \text{Area} \cdot \rho_i$$

The mass in the capillary tube follows

$$M_{cap} = \sum_o^i M_i$$

The density in the cold bulb ρ_c is

$$\rho_c = \frac{M_s - (M_w + M_{cap})}{V_c}$$

Knowing P_m and ρ_c defines a cold temperature T_c in the bulb. The correction for the portion of the capillary tubing having a thermal gradient is especially important in the gas bulb range. If the tube is assumed ambient along its entire length, errors of 30% to 75% can be introduced for temperatures 15°K and above.

INTERSECTION OF GAS AND VAPOR CURVES

Figure 2 shows a plot of P vs. T for two systems. Both systems have exactly the same geometry, but their charge pressures are different. P_I of 5 atm intersects the vapor curve at 4.7°K. P_I of 8 atm intersects at 5.1°K. Raising P_I has increased the useful VPT range significantly. Figure 3 shows a plot of $\frac{dP}{dT}$ vs. T to illustrate the gas bulb temperature sensitivity of the VPT.

In the temperature region of ≈5°K, M_{cap} is 2.3 times that at an assumed T_{charge} = 300°K. Defining the volume ratio R in the vicinity of 5°K as

$$R_{5°K} = \frac{V_w + 2.3V_{cap}}{V_c} = \frac{V_w'}{V_c}$$

is a good approximation and avoids dealing with absolutes. Figure 4 shows a plot of P_I vs. R. The curves represent the locus of points whose P_I and R give intersection temperatures of 4.5, 4.75, 5.0°K. If the geometry of a system is fixed, P_I may be adjusted to optimize the range of the VPT. If P_I is fixed because of gauge resolution considerations, R can be appropriately adjusted.

There are some corrections to Figure 4 when L_c becomes sufficiently large. Figure 5 shows P_I vs. R at $T_{intersection}$ = 5°K for two values of L_c, 1.5 and 7.0 meters. The correction is important in analyzing heat exchangers with large L_c.

EXCESS MASS SYSTEMS

It is possible, through over-charging the system, or making R too large, to preclude the intersection of the gas and vapor bulb curves. A volume ratio of R = 79, charged to P_I = 10 atm demonstrates this effect in Figure 6. The gas in the bulb cools, dropping the system pressure as before. Now, however, it passes above the critical pressure at the critical temperature. At exactly the critical temperature it can be shown that the bulb density at point A is greater than at the C.P. The critical conditions at C.P. require the system at point A to rise in pressure to provide a lower density equal to C.P. Therefore, A is a stable point. Figure 7 shows a temperature-entropy chart for helium where C.P. and A are shown at 5.2°K. Point A at 3.4 atm follows the solid line to the left of the liquid dome, never penetrating it.

Liquid is formed but never in an equilibrium state so it is useless as a VPT. The same arguments can be applied to points B and C to show that B is stable; i.e., has a greater density than point C. The generalization is that the two curves never meet.

This can be thought of as over-filling the bulb. The point B represents a bulb full of subcooled liquid (like a bubble chamber) that cannot exhibit control over the system pressure by condensation. Figure 8 shows P_I vs. R where $T_{intersection} = 5.1°K$. The shaded region represents combinations of P_I and R which cause excess mass problems. No attempt, beyond defining the region of excess mass, is made to quantify this case.

PERCENT LIQUID IN BULB

To prevent over-filling bulbs, and maintain speed of response, it is useful to know the fraction of liquid in the bulb in the VPT range. There are two densities defined by T_c, that of the liquid ρ_l, and that of the gas ρ_g. The fraction of liquid follows:

$$\text{Liq. Frac} = \frac{(M_s - (M_w + M_c)) - V_c \rho_g}{V_c(\rho_l - \rho_g)}$$

If the fraction is less than 50%, the response of the gauge will be improved. VPTs which meet the criteria of Figure 8 will fill to 50% at coldest operating temperatures, thereby making bulb over-fill impossible.

CONCLUSION

The calculations shown can lead to a greater understanding of VPTs, however, because of the number of calculations which must be performed for each change in the system, a computer program has been written to solve these equations. The program is written specifically for helium VPTs and utilizes subroutines for the thermophysical properties of helium-4, developed by R.D.McCarty of the National Bureau of Standards in Boulder, Colorado. The data for all of the Figures shown are results of the program, except Figure 1, which is taken from R.L.Powell and W.A.Blanpied, NBS, and Figure 7, also from NBS. The program is under public file name TEMFIL. By changing the subroutines to properties of other gases, and by making slight modifications, the program can be used to study VPTs with any charging fluid.

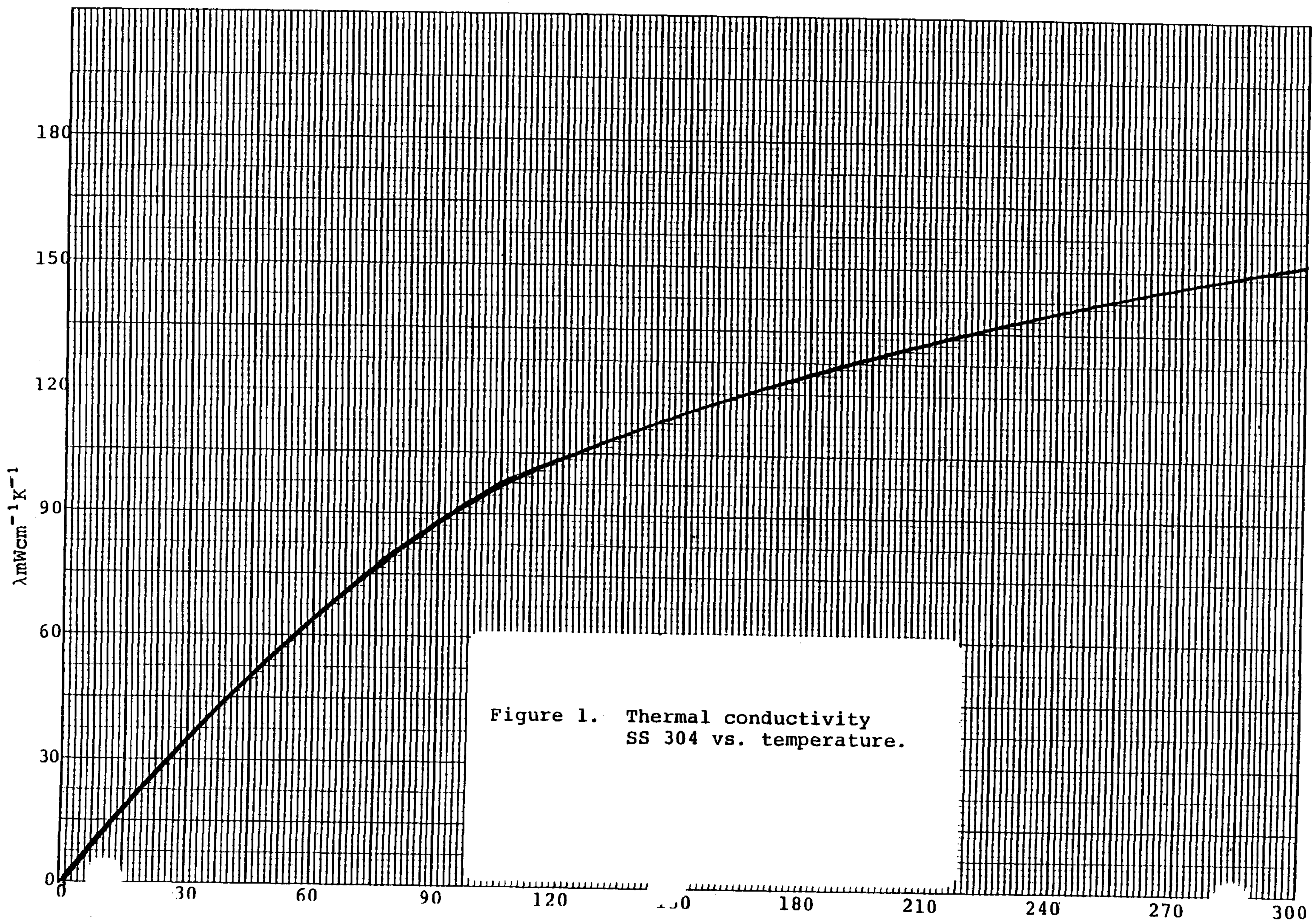

Figure 1. Thermal conductivity SS 304 vs. temperature.

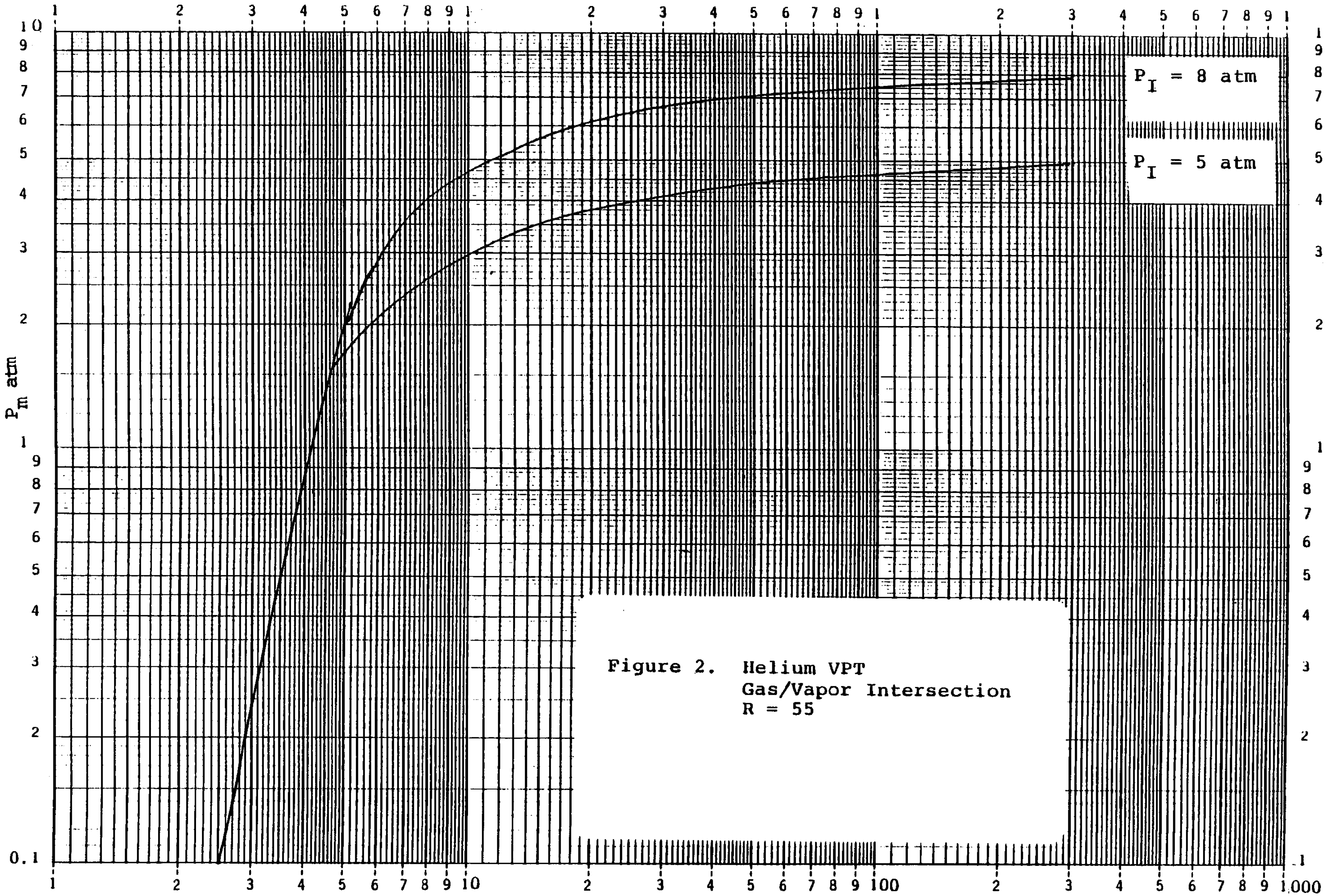

Figure 2. Helium VPT
Gas/Vapor Intersection
R = 55

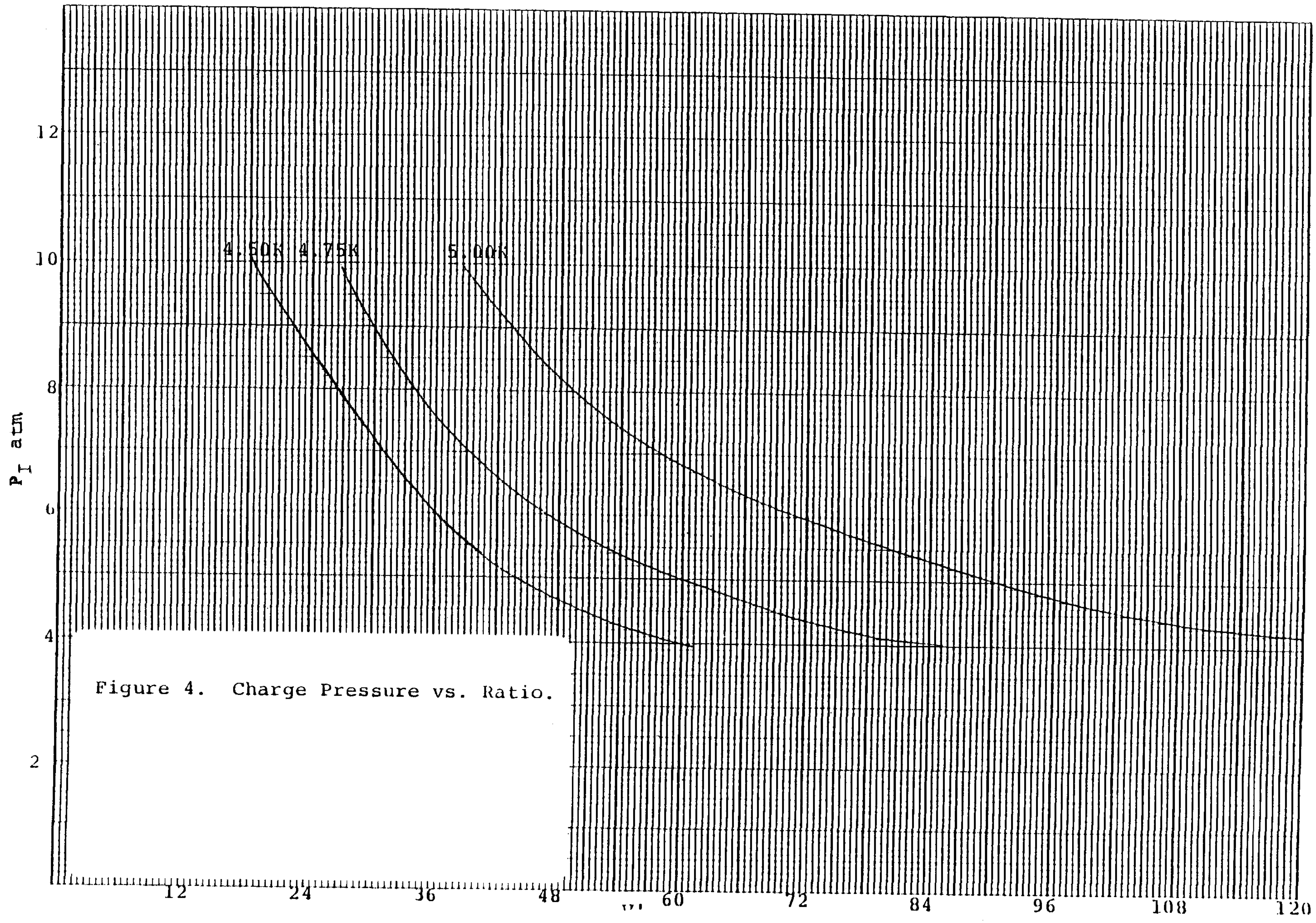

Figure 4. Charge Pressure vs. Ratio.

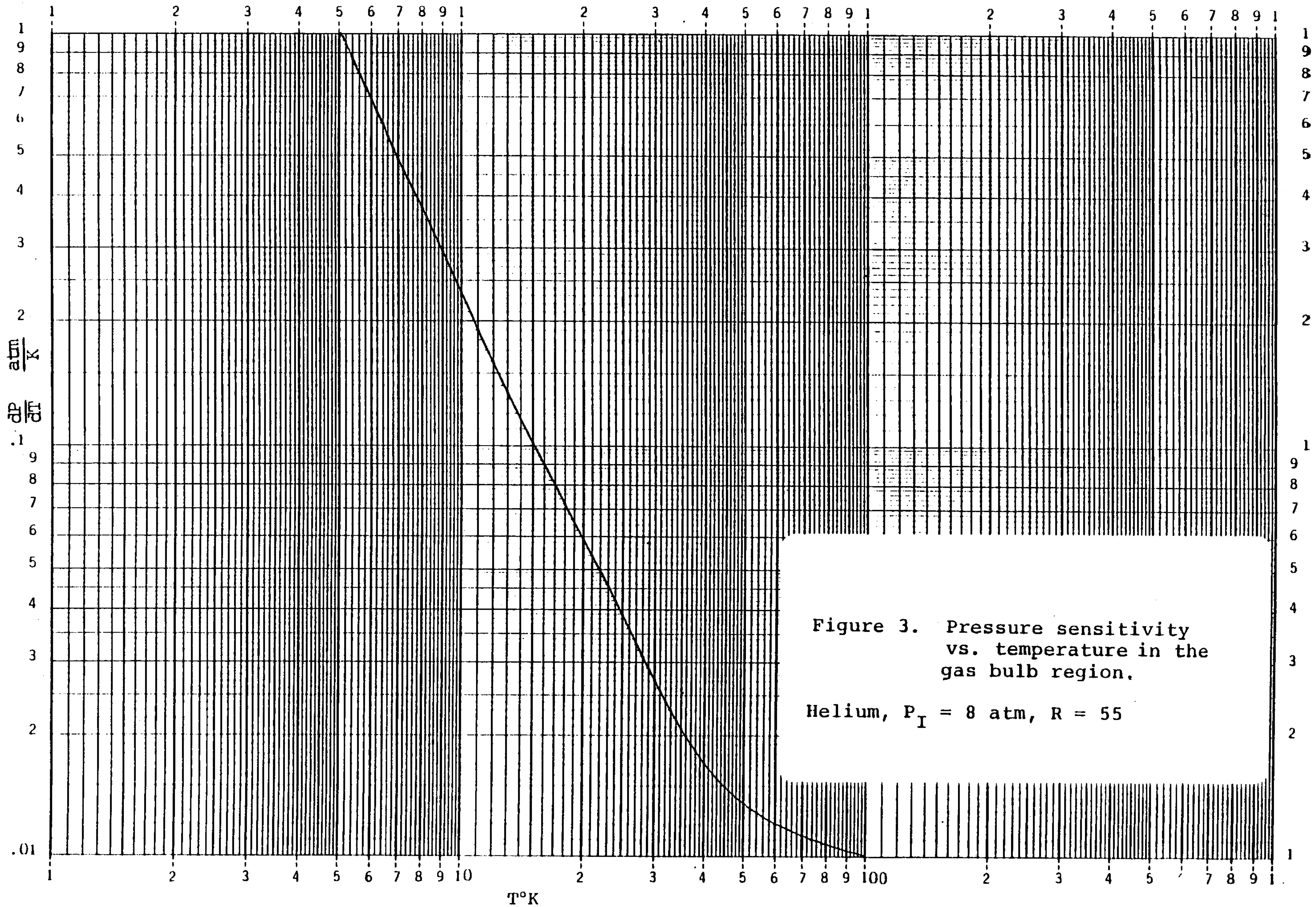

Figure 3. Pressure sensitivity vs. temperature in the gas bulb region.

Helium, P_I = 8 atm, R = 55

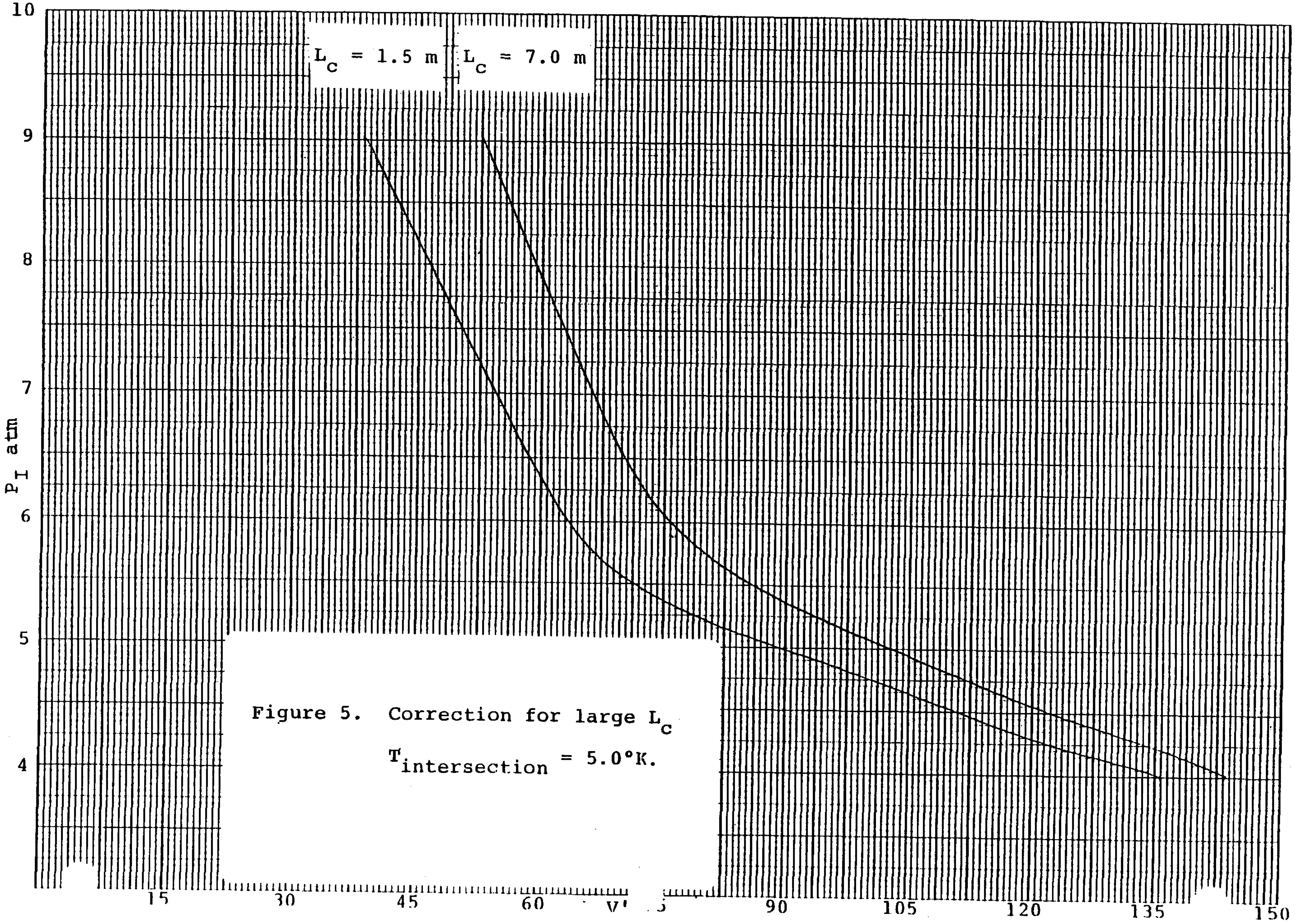

Figure 5. Correction for large L_c

$T_{intersection} = 5.0°K.$

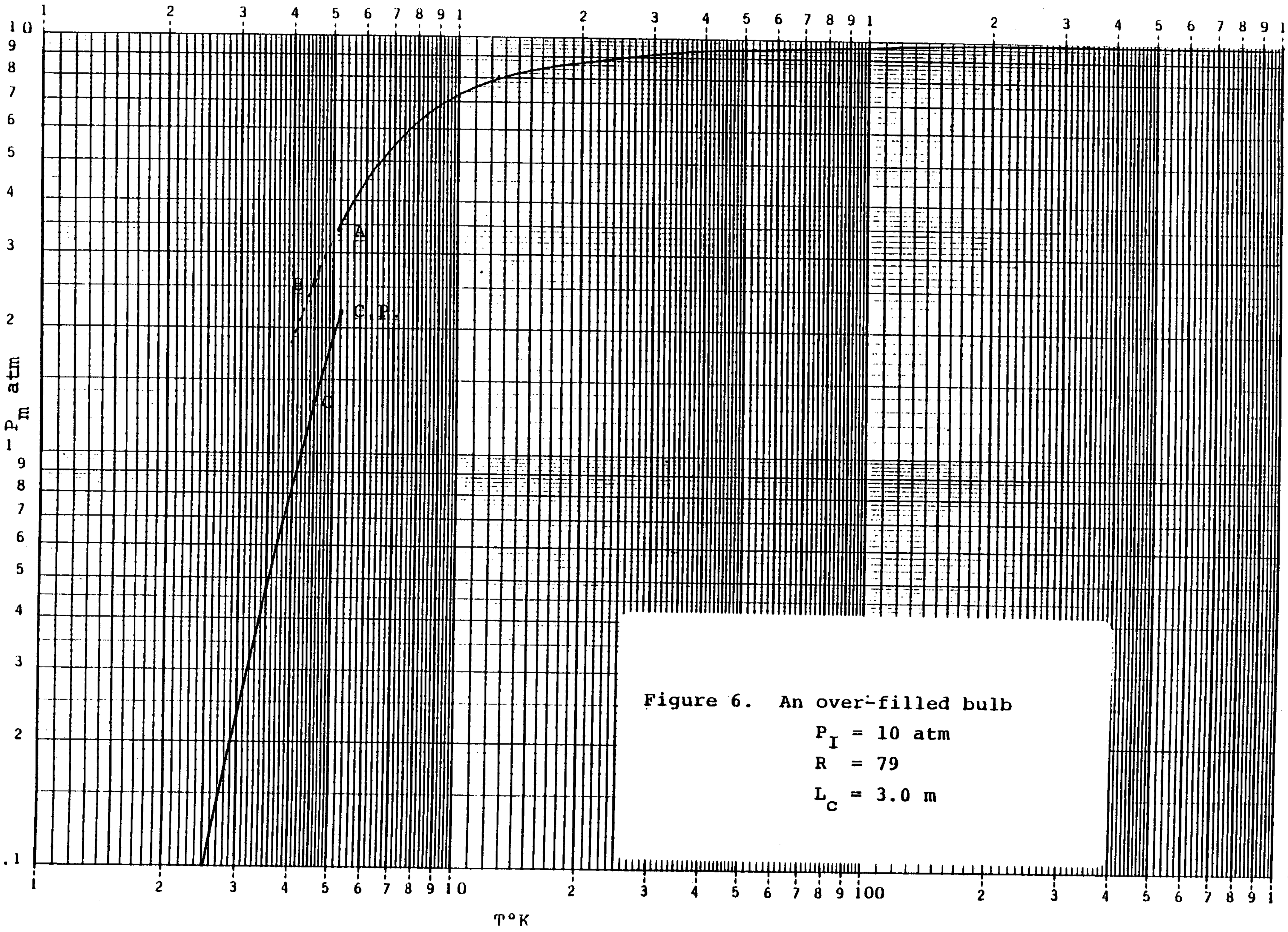

Figure 6. An over-filled bulb

P_I = 10 atm

R = 79

L_C = 3.0 m

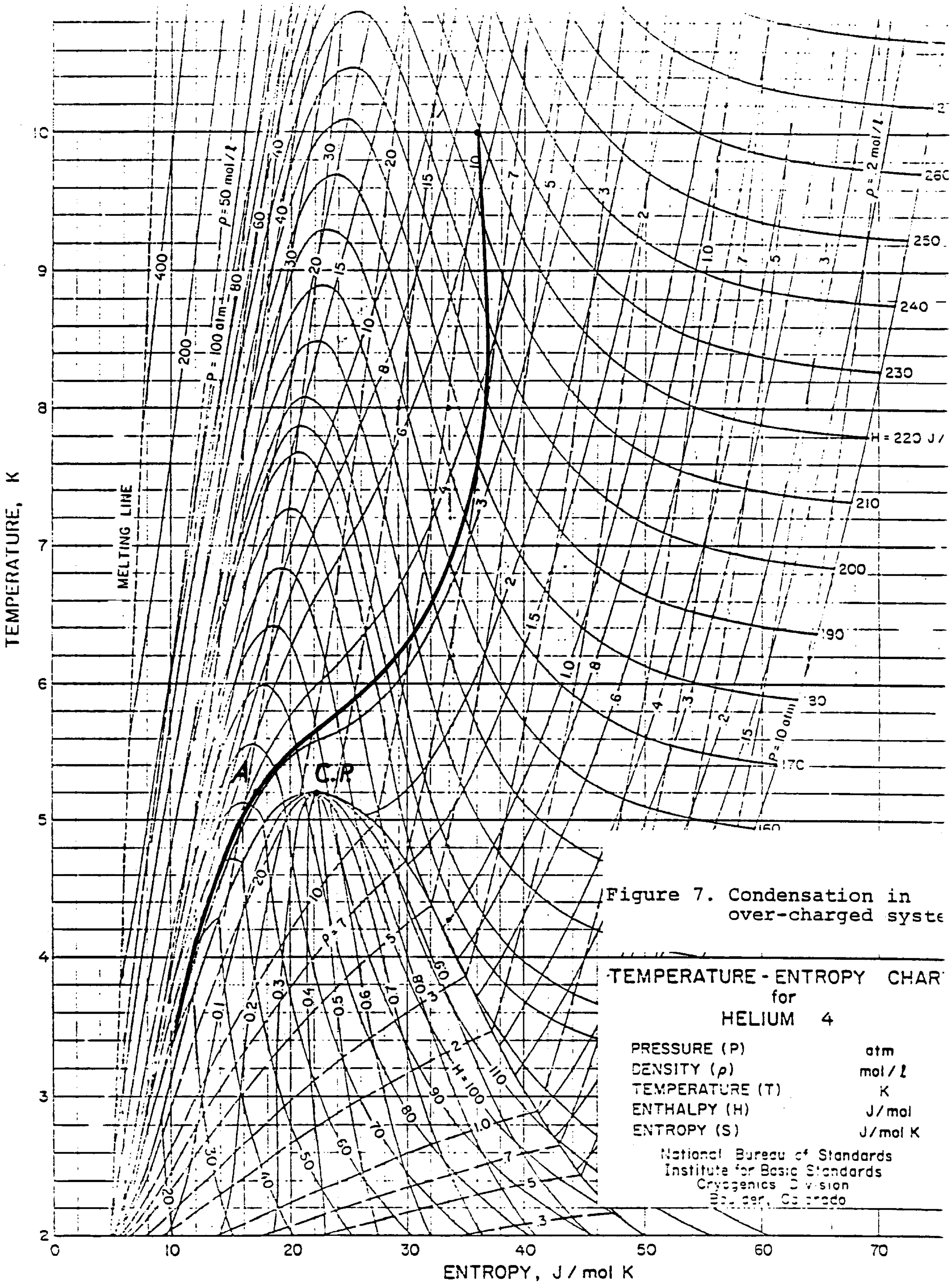

Figure 7. Condensation in over-charged syste

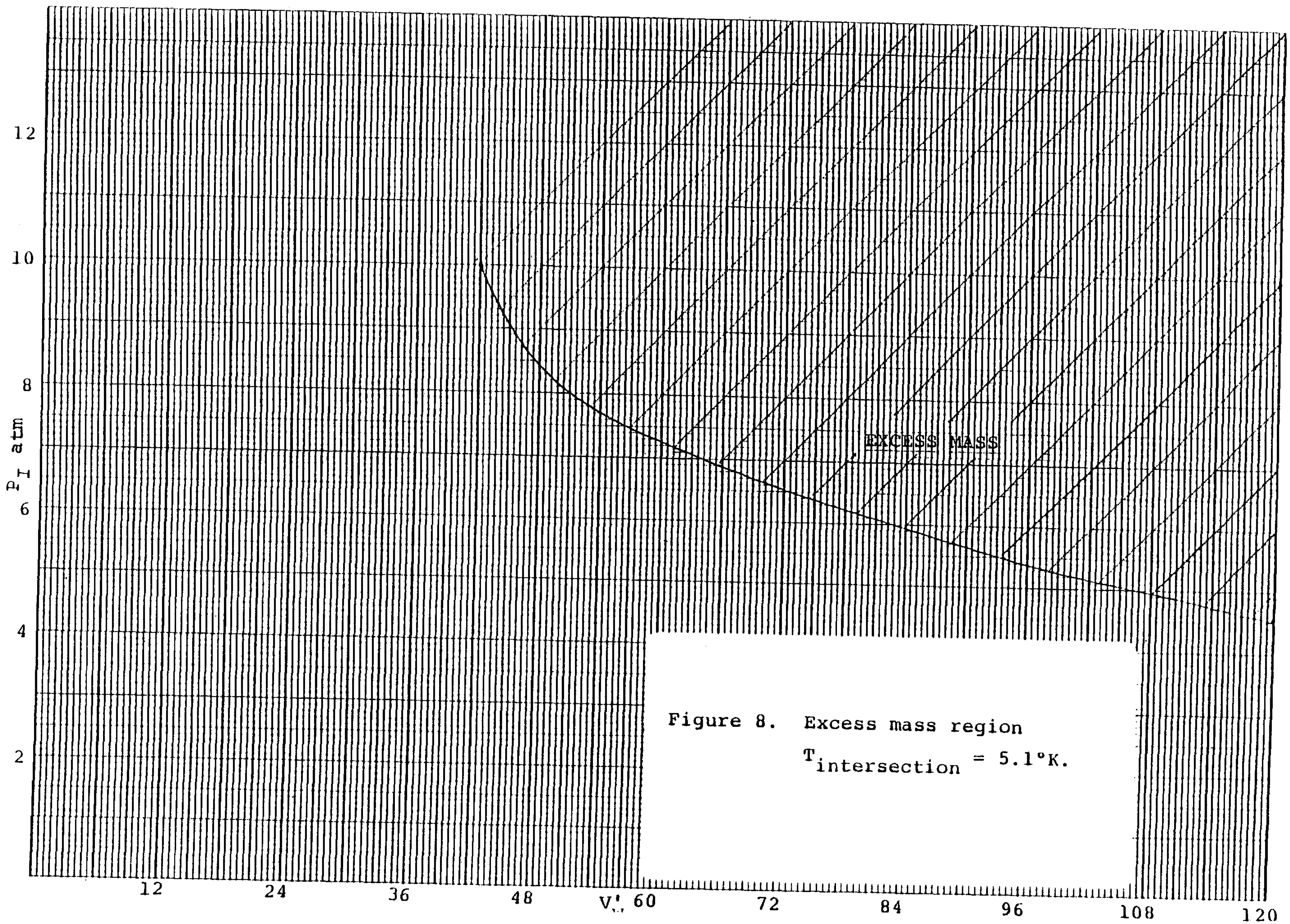

Figure 8. Excess mass region
$T_{intersection} = 5.1°K.$

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FILE/TEMFIL

PROGRAM: HELIUM

ANALYZES HELIUM VPT DESIGN

Initial Parameters:

```
     PC = Initial charge pressure
     PM = Pressure measured from gauge
     CT = Initial charge temperature
     TM = Ambient temperature at time of measurement
     VW = Warm volume, excluding cap tube experiencing
          thermal gradient
 CAPLEN = Length of cap tube experiencing gradient

   DENO = FINDD (PC, CT, DI)
          Finds density in system at initial conditions
          where DI is an initial estimate of density.

  CONST = VT*DENO
          Finds system mass CONST, where VT is total
          system volume.

   DENW = FINDD (PM, TM, DI)
          Finds density in warm region for a given PM and TM.

   DENC = (CONST - VW + VCAP)*DENW)/VC
          Calculates cold density assuming no thermal gradient
          in CAPLEN.

   TEMP = FINDT (PM, DENC)
          Finds temperature in bulb defined by PM and DENC.

 FACTOR = ((-4.33E-3)*TEMP)+2.3
          Finds factor of increase for VCAP which is the mass
          factor of increase in CAPLEN as temperature of bulb
          drops.

  RATIO = (VW + (FACTOR*VCAP))/VC
```

Determines $R = \frac{V_T}{V_C}$

Subroutine CAPMAS

Array contains values of temperatures and their respective thermal conductivities for SS 304 from 300°K to 5°K.

CONDUC = (-1*((-4.17E-7*TEMP)+5E-4)*TEMP*TEMP)+30.6
Determines the integrated value of thermal conductivity vs. temperature from 300°K to TEMP where TEMP is the initial temperature estimate assuming CAPLEN warm.

DELTAT is difference between two temperatures in the array.

AVET is their average temperature.

AVEK is their average value of thermal conductivity.

IF(AVET.LT.TEMP)GOTO 503
Stops loop when it has read through the array down to temperature in bulb.

SEGLEN = AVEK*CAPLEN/CONDUC*DELTAT
Determines L_i for a section of CAPLEN.

DENSEG = P_i for a section of CAPLEN.

SEGMAS = M_i

SUMMAS = $\sum_{o}^{i} M_i$

DENCC = (CONST-(VW*DENW+SUMMAS))/VC
Corrects cold density in bulb now taking into account thermal gradient.

IF (DENCC.LT.0.0) TEMP = TEMP+12

IF (DENCC.LT.0.0) GOTO 400

If initial estimate of TEMP is very low, DENCC can be negative because SUMMAS will have too large a value. This occurs in the gas bulb region where TEMP is usually a low first estimate. These statements correct this situation by increasing TEMP.

TEMPC = FINDT(PM,DENCC)
Corrects initial temperature estimate.
Corrected temperature TEMPC, will be too large if initial estimate TEMP is too low.

```
TEMP = TEMP + 0.2
IF (TEMPC.GT.TEMP) DIFF = TEMPC - TEMP
IF (TEMP.GT.TEMPC) DIFF = TEMP - TEMPC
IF (DIFF.LT.0.4) GOTO 403
```

This series of statements makes TEMP and TEMPC converge to an accuracy of 0.4. For lower temperatures the accuracy can be increased.

Cont'd.

The pressure at TEMPIN follows:

PRESIN = (SLOP1*TEMPIN) + B1

By repeating the procedure until the slopes are equal, or until the intersecting pressure is just greater than the pressure given, the point of intersection can be found.

Subroutine MTEST

This subroutine tests to see if the system mass is too large. If the temperature on the gas curve at 2.2 atm is less than 5.2°K, the gas curve has "over-shot" the vapor curve and an over-massed system exists.

EL:er
8/14/81

```
      PROGRAM HELIUM(INPUT,OUTPUT)
      COMMON VW,VC,CONST,TM,PRESIN,TEMPT,PM,CAPLEN,SUMMAS,TEMP,DENW,
     2 DENCC
C             GIVE INITIAL PARAMETERS
      PRINT 20
      READ*,PC
      PRINT 25
      READ*,PM
      PRINT 30
      READ*,CT
      PRINT 35
      READ*,TM
      PRINT 40
      READ*,VW
      PRINT 45
      READ*,CAPLEN
20    FORMAT(*  GIVE CHARGE PRES ATM *)
25    FORMAT(*  GIVE MEASURED PRES ATM *)
30    FORMAT(*  GIVE CHARGE TEMP K *)
35    FORMAT(*  GIVE MEASURED TEMP K *)
40    FORMAT(*  GIVE  VOL WARM CC   *)
45    FORMAT(* GIVE LENGTH OF CAP TUBING CM  *)
C          FINDS TEMP IN BULB & SYSTEM DENSITIES
C           ASSUMING ALL CAP. TUBE AMBIENT
      VC=.339
C      VOLUME VC SET FOR MOST BULBS AT LAB
C      VOLUME IN CAP TUBE ASSUMES .125 OD ,.049 WALL
      VCAP=CAPLEN*0.0037
      VT=VW+VC+VCAP
      DI=0.0120
      DENO=FINDD(PC,CT,DI)
      CONST=VT*DENO
      DENW=FINDD(PM,TM,DI)
      DENC=(CONST-(VW+VCAP)*DENW)/VC
C        INCREASES VCAP BY FACTOR DEPENDENT ON TEMP AT LOW END
      TEMP=FINDT(PM,DENC)
      FACTOR=((-4.33E-3)*TEMP)+2.3
      RATIO=(VW+(FACTOR*VCAP))/VC
      PRINT37,TEMP,DENC
37    FORMAT(2(F11.6))
      CALL CAPMAS
      PRINT 212
      PRINT 150,TEMP,DENC,DENCC,DENW
C          TESTS TO SEE IF SYSTEM MASS TOO LARGE
      CALL MTEST
      IF(TEMPT.LT.5.2014) GOTO 50
      IF(PM.GT.2.245) GOTO 50
      CALL INTSEC
      IF(PRESIN.LT.PM) GOTO 50
C          CALCULATES PERCENT OF LIQUID IN BULB
      TEMV=VPTEMP(PM)
      DL=DSATL(TEMV)
      DG=DSATV(TEMV)
      WM=VW*DENW+SUMMAS
      CM=CONST-WM
      Y=(CM-(VC*DL))/(DG-DL)
      X=VC-Y
      PERC=(X/VC)*100
      IF(PERC.LT.10) GOTO 50
      PRINT 101
101   FORMAT(*  CHA PRES  MEAS PR  CHA TEM   RATIO    *)
      PRINT 105,PC,PM,CT,RATIO
105   FORMAT(5(2X,F6.2))
      PRINT106
```

```
106     FORMAT(*    PERC LIQ IN BULB    *)
        PRINT 107,PERC
107     FORMAT(F7.3)
        PRINT110
110     FORMAT(*  TEMP       DENCG     DENCL      DENW       *)
        PRINT 240,TEMV,DG,DL,DENW
240     FORMAT(5(F10.6))
        IF(PM.LT.2.245) GOTO 200
50      PRINT 208
208     FORMAT(*  YOU ARE IN THE GAS BULB RANGE  *)
        PRINT 210
210     FORMAT(*  CHA PRES MEAS PR  CHA TEM   RATIO  *)
        PRINT 211,PC,PM,CT,RATIO
211     FORMAT(5(2X,F6.2))
C        PRINT 212
212     FORMAT(*  TEMP      DENC  DENCC     DENW  *)
C        PRINT 150,TEMP,DENC,DENCC,DENW
150     FORMAT(4(F10.6))
200     CONTINUE
        STOP
        END
C          SUBROUTINE TO TEST SYSTEM MASS
        SUBROUTINE MTEST
        COMMON VW,VC,CONST,TM,PRESIN,TEMPT,PM,CAPLEN,SUMMAS,TEMP,DENW,
     2  DENCC
C        LOOKS AT GAS CURVE AT 2.2 ATM TO SEE IF IT HAS
C        OVER SHOT THE VAPOR CURVE.
        DI=.012
        PRES=2.2
        DENWAR=FINDD(PRES,TM,DI)
        DENCOL=(CONST-(VW*DENWAR+SUMMAS))/VC
        TEMPT=FINDT(PRES,DENCOL)
        IF(TEMPT.LT.5.2014) PRINT 155
155     FORMAT(*  MASS TOO LARGE  *)
        RETURN
        END
C             SUBROUTINE TO CALCULATE MASS IN CAP TUBE
C             EXPERIENCING THERMAL GRADIENT
        SUBROUTINE CAPMAS
        COMMON VW,VC,CONST-TM,PRESIN,TEMPT,PM,CAPLEN,SUMMAS,TEMP,DENW,
     2  DENCC
C        ARRAY CONTAINS TEMPS AND THERMAL CONDUC. VALUES FOR
C        SS 304.  EXTRAPOLATED DATA PTS FROM FIG. #1
        DIMENSION ARRAY(2,66)
        DATA ARRAY/300.0,.152,294.6,.151,289.2,.150,283.8,.149,
     2  278.4,.148,273.0,.147,268.4,.146,263.8,.145,259.2,.144,
     3  254.6,.143,250.0,.142,241.6,.141,233.3,.139,225.0,.136,
     4  216.6,.134,208.3,.132,200.0,.130,193.3,.128,186.7,.126,
     5  180.0,.124,173.4,.122,166.7,.120,160.0,.188,153.4,.116,
     6  146.7,.114,136.7,.110,126.7,.106,116.7,.102,106.7,.098,
     7  96.7,.094,90.0,.090,82.9,.086,77.0,.082,74.0,.078,
     8  70.0,.074,65.0,.070,60.0,.066,55.0,.062,50.0,.058,
     9  46.0,.054,42.0,.050,39.0,.046,36.0,.042,33.0,.038,
     *  31.0,.034,28.0,.030,25.0,.027,24.0,.026,23.0,.025,
     *  22.0,.023,21.0,.022,20.0,.021,19.0,.020,18.0,.018,
     *  17.0,.017,16.0,.016,15.0,.015,14.0,.014,13.0,.012,
     *  11.0,.010,10.0,.0085,9.0,.0075,8.0,.0065,7.0,.0055,
     *  6.0,.0045,5.0,.0034/
        DO 400 K=1,500
C        PRINT 401,TEMP,DENCC,TEMPC
C         DETERMINES THE INTEGRATED VALUE OF THERM. COND.
C         FROM 300K TO BULB TEMP
        CONDUC=(-1*((-4.17E-7*TEMP)+5E-4)*TEMP*1   P)+30.6
        SUMMAS=0.0
```

```
60        CONTINUE
C          PRINT120
120       FORMAT(* SLOPE#1    PRES    TEMP       *)
C          PRINT 250,SLOP1,PR,TEMP,B1
C          PRINT130
130       FORMAT(* SLOPE#2    PRES    TEMP       *)
C          PRINT 250,SLOP2,PR,TEM,B2
          PRINT140
140       FORMAT(*INT TEMP   INT PRES*)
          PRINT 270,TEMPIN,PRESIN
          IF(PRESIN.GT.2.245) PRINT 119
119       FORMAT(*   INT. PRES ABOVE CRIT. PRES   *)
230       FORMAT(6(2X,F6.2))
250        FORMAT(3(F9.4))
270        FORMAT(2(F9.4))
          RETURN
          END
C             FINDS TEMP AT KNOWN PRESS. & DENSITY
          FUNCTION FINDT(P,D)
C                        CRITICAL PRESSURE.
          DATA PC /2.245/
C                        SUPPLY INITIAL TEMPERATURE
C                        AND DENSITY ESTIMATES.
          TO=250.0
          DI = 0.120
C
C                        FIND 2-PHASE TEMPERATURE.
          TV = 2.0
          IF(P.LT.PC) TV = VPTEMP(P)
          DO 100 ITER=1,20
          DO=FINDD(P,TO,DI)
          DP=FINDD(P,TO+0.1,DI)
          DDDT=(DP-DO)/0.1
          DELTA=D-DO
          IF(DDDT.EQ.0.0) DDDT=DELTA
          TO=0.25*DELTA/DDDT+TO
          IF(TO.LT.TV) TO = TV
C            PRINT 200, DI,DO,DP,DDDT,DELTA,TO,P,D
200       FORMAT (8E9.3)
100       DI = DO
          FINDT=TO
          RETURN
          END
```

```
C         SUMS MASS IN CAP TUBE EXPERIENCING GRADIENT
          DO 500 I=2,66
          DELTAT=ARRAY(1,I-1)-ARRAY(1,I)
          AVET=(ARRAY(1,I-1)+ARRAY(1,I))/2
          AVEK=(ARRAY(2,I-1)+ARRAY(2,I))/2
          IF(AVET.LT.TEMP) GOTO 503
          AH=.0037
          SEGLEN=AVEK*CAPLEN/CONDUC*DELTAT
          DI=.012
          DENSEG=FINDD(PM,AVET,DI)
          SEGMAS=DENSEG*SEGLEN*AH
          SUMMAS=SUMMAS+SEGMAS
C          PRINT 502,SUMMAS,SEGLEN,AVEK,AVET,DELTAT,CONDUC
502       FORMAT (6(F12.5))
500       CONTINUE
503       CONTINUE
C          PRINT 505,SUMMAS
505       FORMAT(E13.4)
C               CORRECTS COLD DENSITY AND CORRECTS TEMP
          DENCC=(CONST-(VW*DENW+SUMMAS))/VC
          IF(DENCC.LT.0.0) TEMP=TEMP+12
          IF(DENCC.LT.0.0) GOTO 400
          TEMPC=FINDT(PM,DENCC)
          TEMP=TEMP+0.2
          IF(TEMPC.GT.TEMP) DIFF=TEMPC-TEMP
          IF(TEMP.GT.TEMPC) DIFF=TEMP-TEMPC
          IF(DIFF.LT.0.4) GOTO 403
401       FORMAT(3(E13.4))
400       CONTINUE
403       CONTINUE
          RETURN
          END
C            SUBROUTINE TO FIND INTERSECTION OF VPT & GAS CURVES
          SUBROUTINE INTSEC
          COMMON VW,VC,CONST,TM,PRESIN,TEMPT,PM,CAPLEN,SUMMAS,TEMP,DENW,
        2 DENCC
C                   TAKES DERIVATIVE AT PT. ON GAS CURVE
          PR=2.2
30        DI=0.012
          DENW1=FINDD(PR,TM,DI)
          DENC1=(CONST-(VW*DENW1+SUMMAS))/VC
          TEMP1=FINDT(PR,DENC1)
          PR2=PR-.1
          DENW2=FINDD(PR2,TM,DI)
          DENC2=(CONST-(VW*DENW2+SUMMAS))/VC
          TEMP2=FINDT(PR2,DENC2)
          SLOP1=0.1/(TEMP1-TEMP2)
          IF(SLOP1.LT.0)SLOP1=-1*SLOP1
          B1=PR-(SLOP1*TEMP1)
C                 FINDS TEMP ON VAPOR CURVE & TAKES DERIVATIVE
          TEM1=VPTEMP(PR)
          PR2=PR-0.1
          TEM2=VPTEMP(PR2)
          SLOP2=0.1/(TEM1-TEM2)
          B2=PR-(SLOP2*TEM1)
C               FINDS INTERSECTION OF VPT AND GAS CURVES
          IF(SLOP1.EQ.SLOP2)TEMPIN=TEM1
          IF(SLOP1.EQ.SLOP2)PRESIN=PR
          IF(SLOP1.EQ.SLOP2) GOTO 60
          TEMPIN=-1*(B1-B2)/(SLOP1-SLOP2)
          PRESIN=(SLOP1*TEMPIN)+B1
          IF(PRESIN.GT.PR-.01) GOTO 60
          IF(PRESIN.LT.PR)PR=((PR-PRESIN)/2)+PRESIN
          GOTO 30
```